1. Volume of a Cylinder The volume V of a right circular cylinder of height h and radius r is $V = \pi r^2 h$. If the height is twice the radius, express the volume V as a function of r.

Context - 1pt, show work - 1pt, Final - 1pt

1. $V = \pi r^2 h,\ \ h = 2r \Rightarrow V(r) = \pi r^2 \cdot (2r) = 2\pi r^3$

3. **Demand Equation** The price p, in dollars, and the quantity x sold of a certain product obey the demand equation

$$p = -\frac{1}{6}x + 100, \qquad 0 \le x \le 600$$

(a) Express the revenue R as a function of x. (Remember, $R = xp$.)
(b) What is the revenue if 200 units are sold?
(c) Graph the revenue function using a graphing utility.
(d) What quantity x maximizes revenue? What is the maximum revenue?
(e) What price should the company charge to maximize revenue?

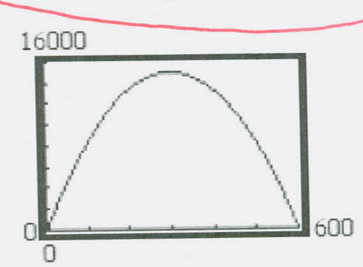
3. a. $R(x) = x\left(-\frac{1}{6}x + 100\right) = -\frac{1}{6}x^2 + 100x$

b. $R(200) = -\frac{1}{6}(200)^2 + 100(200)$

$= \frac{-20{,}000}{3} + 20{,}000$

$= \frac{40{,}000}{3} \approx \$13{,}333.33$

c.



1pt - Context
1pt - Show work
1pt - Final answer

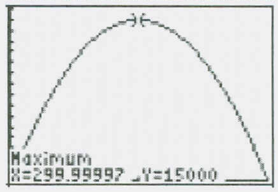
d. $x = 300$ maximizes revenue

$R(300) = -\frac{1}{6}(300)^2 + 100(300)$

$= -15{,}000 + 30{,}000$

$= \$15{,}000$

The maximum revenue is \$15,000.



e. $p = -\frac{1}{6}(300) + 100 = -50 + 100 = \$50$

maximizes revenue

7. Enclosing a Rectangular Field David has available 400 yards of fencing and wishes to enclose a rectangular area.
(a) Express the area A of the rectangle as a function of the width x of the rectangle.
(b) What is the domain of A?
(c) Graph $A = A(x)$ using a graphing utility. For what value of x is the area largest?

1pt

7pts

7. a. Let x = width and y = length of the rectangular area. 1pt

$P = 2x + 2y = 400$

$$y = \frac{400 - 2x}{2} = 200 - x$$ 1pt

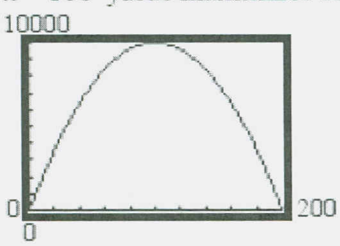
Then

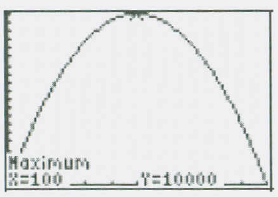
$$A(x) = (200 - x)x$$
$$= 200x - x^2$$
$$= -x^2 + 200x$$ 1pt

b. We need $x > 0$ and $y > 0$. This means $200 - x > 0 \Rightarrow 200 > x$.

Thus, the domain of A is $\{x | 0 < x < 200\}$.

c. $x = 100$ yards maximizes area





9. Let $P = (x, y)$ be a point on the graph of $y = x^2 - 8$.
(a) Express the distance d from P to the origin as a function of x.
(b) What is d if $x = 0$?
(c) What is d if $x = 1$?
(d) Use a graphing utility to graph $d = d(x)$.
(e) For what values of x is d smallest?

9. a. The distance d from P to the origin is $d = \sqrt{x^2 + y^2}$. Since P is a point on the graph of $y = x^2 - 8$, we have:

$$d(x) = \sqrt{x^2 + (x^2 - 8)^2} = \sqrt{x^4 - 15x^2 + 64}$$ 1pt

b. $d(0) = \sqrt{0^4 - 15(0)^2 + 64} = \sqrt{64} = 8$

c. $d(1) = \sqrt{(1)^4 - 15(1)^2 + 64}$
$= \sqrt{1 - 15 + 64} = \sqrt{50} = 5\sqrt{2} \approx 7.07$

10. Let $P = (x, y)$ be a point on the graph of $y = x^2 - 8$.
(a) Express the distance d from P to the point $(0, -1)$ as a function of x.
(b) What is d if $x = 0$?
(c) What is d if $x = -1$?
(d) Use a graphing utility to graph $d = d(x)$.
(e) For what values of x is d smallest?

1pt

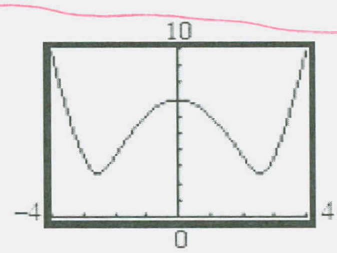
10. a. The distance d from P to $(0, -1)$ is $d = \sqrt{x^2 + (y+1)^2}$. Since P is a point on the graph of $y = x^2 - 8$, we have: 1pt

$$d(x) = \sqrt{x^2 + (x^2 - 8 + 1)^2}$$
$$= \sqrt{x^2 + (x^2 - 7)^2} = \sqrt{x^4 - 13x^2 + 49}$$

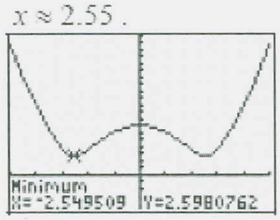
b. $d(0) = \sqrt{0^4 - 13(0)^2 + 49} = \sqrt{49} = 7$ 1pt

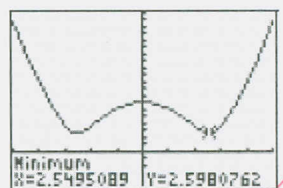
c. $d(-1) = \sqrt{(-1)^4 - 13(-1)^2 + 49} = \sqrt{37} \approx 6.08$

d.



e. d is smallest when $x \approx -2.55$ and when $x \approx 2.55$.

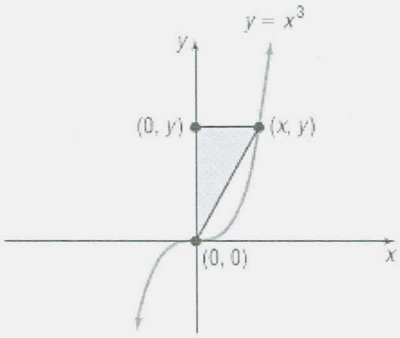




Bonus 2pts

9pts

13. A right triangle has one vertex on the graph of $y = x^3$, $x > 0$, at (x, y), another at the origin, and the third on the positive y-axis at $(0, y)$, as shown in the figure. Express the area A of the triangle as a function of x.

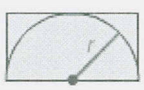


All good test questions!

13. By definition, a triangle has area $A = \frac{1}{2}bh$, b = base, h = height. From the figure, we know that $b = x$ and $h = y$. Expressing the area of the triangle as a function of x, we have:

$$A(x) = \frac{1}{2}xy = \frac{1}{2}x\left(x^3\right) = \frac{1}{2}x^4.$$

23. A semicircle of radius r is inscribed in a rectangle so that the diameter of the semicircle is the length of the rectangle (see the figure).



(a) Express the area A of the rectangle as a function of the radius r of the semicircle.
(b) Express the perimeter p of the rectangle as a function of r.

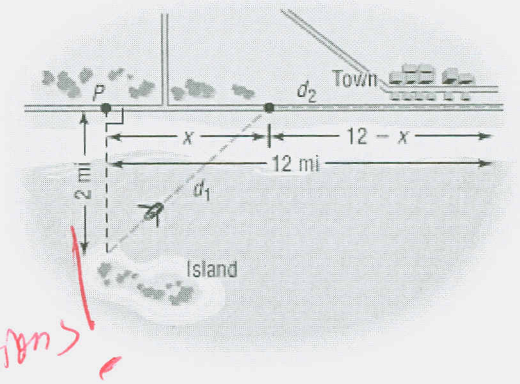
23. a. A = area, r = radius; diameter $= 2r$

$$A(r) = (2r)(r) = 2r^2$$

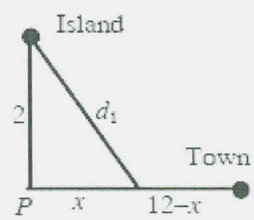
b. p = perimeter

$$p(r) = 2(2r) + 2r = 6r$$

31. Time Required to Go from an Island to a Town An island is 2 miles from the nearest point P on a straight shoreline. A town is 12 miles down the shore from P. See the illustration.



31. a. The time on the boat is given by $\frac{d_1}{3}$. The time on land is given by $\frac{12-x}{5}$.



$$d_1 = \sqrt{x^2 + 2^2} = \sqrt{x^2 + 4}$$

The total time for the trip is:

$$T(x) = \frac{12-x}{5} + \frac{d_1}{3} = \frac{12-x}{5} + \frac{\sqrt{x^2+4}}{3}$$

b. Domain: $\left\{ x \mid 0 \le x \le 12 \right\}$

c. $$T(4) = \frac{12-4}{5} + \frac{\sqrt{4^2+4}}{3} = \frac{8}{5} + \frac{\sqrt{20}}{3} \approx 3.09 \text{ hours}$$

d. $$T(8) = \frac{12-8}{5} + \frac{\sqrt{8^2+4}}{3} = \frac{4}{5} + \frac{\sqrt{68}}{3} \approx 3.55 \text{ hours}$$